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# Inverse Scattering of Buried Complex Object by TE Wave Illumination

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**Abstract**

The inverse scattering of buried complex object by transverse electric (TE) wave illumination is investigated. Dielectric cylinders coated on a conductor of unknown permittivities are buried in one half space and scatter a group of unrelated TE waves incident from another half space where the scattered field is recorded. By proper arrangement of the various unrelated incident fields, the difficulties of ill-posedness and nonlinearity are circumvented, and the permittivity distribution can be reconstructed through simple matrix operations. The algorithm is based on the moment method and the unrelated illumination method. Good reconstruction is obtained even in the presence of additive Gaussian random noise in measured data. In addition, the effect of noise on the reconstruction result is also investigated.

## I .INTRODUCTION

In the last few years, electromagnetic inverse scattering problems of underground objects have been a growing importance in many different fields of applied science, with a large potential impact on geosciences and remote sensing applications. Most microwave inverse scattering algorithms developed are for TM wave illuminations in which the vectorial problem can be simplified to a scalar one. On the other hand, much fewer works have been reported on the more complicated TE case [1]-[3]. In the TE wave excitation case, the presence of polarization charges makes the inverse problem more nonlinear. As a result, the reconstruction becomes more difficult. However, the TE polarization case is useful because it provides additional information about the object.

In this paper, the inverse scattering of buried complex object by TE wave illumination is investigated. An efficient algorithm is proposed to reconstruct the permittivity distribution of the objects by using only the scattered field measured outside. The algorithm is based on the unrelated illumination method [4], [5]. In Section II, the theoretical formulation for electromagnetic inverse scattering is presented. Numerical results for objects of different permittivity distributions are given in Section III. Finally, conclusions are drawn in Section IV.

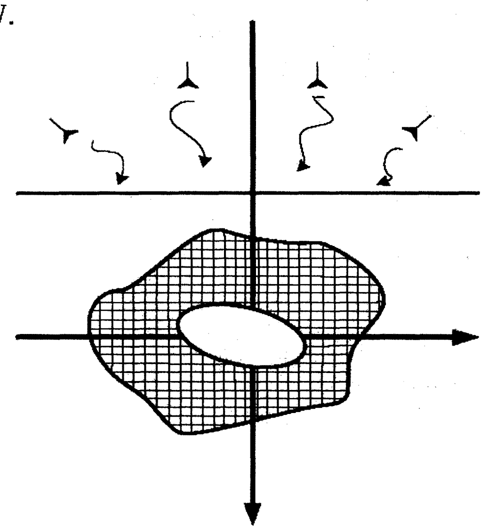

**Fig. 1** Geometry of problem in the (x,y)

## II .THEORETICAL FORMULATION

Let us consider dielectric cylinders buried in a lossless homogeneous half-space as shown in Fig. 1. Media in regions 1 and 2 are characterized by permittivities $\varepsilon_1$ and $\varepsilon_2$, respectively. The permeability is $\mu_0$ for all material including the scatterers. The axis of the buried cylinder is the z-axis; that is, the properties of the scatterer may vary with the transverse coordinates only. A group of unrelated incident wave with magnetic field parallel to the z-axis (i.e., transverse electric, or TE, polarization) is illuminated upon the scatterers. Owing to the interface between region 1 and 2, the incident waves generate two waves that would exist in the absence of the scatterer: reflected waves (for $y \le -a$) and transmitted waves (for $y > -a$). Let the unperturbed field be represented by

$$\overline{E}^i(x,y) = \begin{cases} (E_x^i)_1(x,y)\hat{x} + (E_y^i)_1(x,y)\hat{y}, & y \le -a \\ (E_x^i)_2(x,y)\hat{x} + (E_y^i)_2(x,y)\hat{y}, & y > -a \end{cases}$$

-------------------------------------------------(1)

By using the vector potential techniques, the internal total electric field

$$\bar{E}(x,y) = E_x(x,y)\hat{x} + E_y(x,y)\hat{y}$$

and the external scattered field,

$$\bar{E}^s(x,y) = E_x^s(x,y)\hat{x} + E_y^s(x,y)\hat{y}$$

can be expressed by the following equations:

$$E_x(\bar{r}) = -\left(\frac{\partial^2}{\partial x^2} + k_2^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_x(\bar{r}')ds'\right]$$

$$-\frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_y(\bar{r}')ds'\right]$$

$$-\frac{\partial}{\partial y}\left[\int_c G(\bar{r},\bar{r}')J_{sm}dl'\right] + E_x^i(\bar{r}) \quad \text{-------(2)}$$

$$E_y(\bar{r}) = -\frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_x(\bar{r}')ds'\right]$$

$$-\left(\frac{\partial^2}{\partial y^2} + k_2^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_y(\bar{r}')ds'\right]$$

$$+\frac{\partial}{\partial x}\int_c G(\bar{r},\bar{r}')J_{sm}dl' + E_y^i(\bar{r}) \quad \text{-------(3)}$$

$$E_x^s(\bar{r}) = -\left(\frac{\partial^2}{\partial x^2} + k_2^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_x(\bar{r}')ds'\right]$$

$$-\frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_y(\bar{r}')ds'\right]$$

$$-\frac{\partial}{\partial y}\left[\int_c G(\bar{r},\bar{r}')J_{sm}dl'\right] \quad \text{--------(4)}$$

$$E_y^s(\bar{r}) = -\frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_x(\bar{r}')ds'\right]$$

$$-\left(\frac{\partial^2}{\partial y^2} + k_2^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_r(\bar{r}')-1)E_y(\bar{r}')ds'\right]$$

$$+\frac{\partial}{\partial x}\int_c G(\bar{r},\bar{r}')J_{sm}dl' \quad \text{----------(5)}$$

Here $k_i$ denotes the wave number in region i and $\varepsilon_r$ is the relative permittivity of the dielectric objects. $J_{sm}$ is the equivalent magnetic surface current density in the z direction. $G(x,y;x',y')$ is the half space Green's function, which can be obtained by the Fourier transform .

Since the tangential components of $\bar{E}$ on the surface of the perfect conductor should be zero, one can obtain the following integral equation:

$$-\hat{n}\times[E_x^i(\bar{r})\hat{x} + E_y^i(\bar{r})\hat{y}] = \hat{n}\times[E_x^s(\bar{r})\hat{x} + E_y^s(\bar{r})\hat{y}]$$

-------------------------------------------------(6)

where $\hat{n}$ is the outward unit vector normal to the surface of the conductor.

The direct scattering problem is to calculate the scattered field $E_s$ in region 1, while the permittivity distribution of the buried objects is given. This can be achieved by using (2), (3), and (6) to solve the total field inside the materials $\bar{E}$, the equivalent magnetic surface current density $J_{sm}$ and then calculating $\bar{E}^s$ by (4) and (5). For numerical implementation of the direct problem, the dielectric objects are divided into $N_1$ sufficient small cells. Thus the permittivity and the total field within each cell can be taken as constants. Similarly, the contour of the conductor is divided into $N_2$ sufficient segments. Thus the equivalent magnetic surface current density over each segment can be taken as constants. Then the moment method is used to solve (2)-(6) with a pulse basis function for expansion and point matching for testing. Then (2)-(6) can be transformed into matrix equations

$$\begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} = \left\{\begin{bmatrix} [G_3] & [G_4] \\ [G_4] & [G_6] \end{bmatrix}\begin{bmatrix} [\tau] & 0 \\ 0 & [\tau] \end{bmatrix} + \begin{bmatrix} [I] & 0 \\ 0 & [I] \end{bmatrix}\right\}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$+\begin{pmatrix} [G_5] \\ [G_7] \end{pmatrix}(J_{sm}) \quad \text{-------------------------(7)}$$

$$-(E_h^i) = [G_8][\tau] + [G_9][\tau] + [G_{10}](J_{sm}) \text{---(8)}$$

$$\begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} = \left\{-\begin{bmatrix} [G_{11}] & [G_{12}] \\ [G_{12}] & [G_{14}] \end{bmatrix}\begin{bmatrix} [\tau] & 0 \\ 0 & [\tau] \end{bmatrix}\right\}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$+\begin{pmatrix} [G_{13}] \\ [G_{15}] \end{pmatrix}(J_{sm}) \quad \text{------------------(9)}$$

For the inverse scattering problem, the permittivity distribution of the dielectric objects is to be computed by the knowledge of the scattered field measured in region 1. To solve this problem, the $(J_{sm})$ in (8) is first computed and substituted into (7) and (9). Next we use $2N_1$ different incident column vectors are used to illuminate the object, the follow equations are obtained:

$$[E_t^i] = [[G_{t1}][\tau_t] + [I_t]] \ [E_t] \quad \text{---------(10)}$$

$$[E_t^s] = -[G_{t2}]\ [\tau_t]\ [E_t] \quad \text{------------(11)}$$

where

$$[E_t^i] = \begin{bmatrix} E_x^i + [G_5][G_{10}]^{-1}(E_h^i) \\ E_y^i + [G_7][G_{10}]^{-1}(E_h^i) \end{bmatrix} \quad [E_t] = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$[E_t^s] = \begin{bmatrix} E_x^s + [G_{13}][G_{10}]^{-1}(E_h^i) \\ E_y^s + [G_{15}][G_{10}]^{-1}(E_h^i) \end{bmatrix}$$

$$[G_{t1}] = \begin{bmatrix} [G_3]-[G_5][G_{10}]^{-1}[G_8] & [G_4]-[G_5][G_{10}]^{-1}[G_9] \\ [G_4]-[G_7][G_{10}]^{-1}[G_8] & [G_6]-[G_7][G_{10}]^{-1}[G_9] \end{bmatrix}$$

$$[G_{t2}] = \begin{bmatrix} [G_{11}]+[G_{13}][G_{10}]^{-1}[G_8] & [G_{12}]+[G_{13}][G_{10}]^{-1}[G_9] \\ [G_{12}]+[G_{15}][G_{10}]^{-1}[G_8] & [G_{14}]+[G_{15}][G_{10}]^{-1}[G_9] \end{bmatrix}$$

$$[\tau_t] = \begin{bmatrix} [\tau] & 0 \\ 0 & [\tau] \end{bmatrix} \quad , \ [I_t] = \begin{bmatrix} [I] & 0 \\ 0 & [I] \end{bmatrix}$$

Here $[E_t^i]$ and $[E_t]$ are both $2N_1 \times 2N_1$ matrices. $[E_t^s]$ is a $M \times 2N_1$ matrix. Note that the matrix $[G_8]$ is diagonally dominant and always invertible. It is worth mentioning that other than matrix $[G_{t2}]$, the matrix $[G_{t1}][\tau_t]+[I_t]$ is always a well-posed one in any case, therefore we can first solve $[E_t^i]$ in (10) and substitute into (11), then $[\tau_t]$ can be found by the following equation

$$[\Psi_t][\tau_t] = [\Phi_t] \quad \text{-------------(12)}$$

where

$$[\Phi_t] = -[E_t^s][E_t^i]^{-1}$$

$$[\Psi_t] = [E_t^s][E_t^i]^{-1}[G_{t1}] + [G_{t2}]$$

From (12), all the diagonal elements in the matrix $[\tau]$ can be determined by comparing the element with the same subscripts which may be any row of both $[\Psi_t]$ and $[\Phi_t]$:

$$(\tau)_{nn} = \frac{(\Phi_t)_{mn}}{(\Psi_t)_{mn}}, \quad n \le N \quad \text{----------------(13a)}$$

or

$$(\tau)_{(n-N_1)(n-N_1)} = \frac{(\Phi_t)_{mn}}{(\Psi_t)_{mn}}, \quad n \ge N+1 \text{-------(13b)}$$

Then the permittivities of each cell can be obtained as follows:

$$\varepsilon_n = (\tau)_{nn} + 1 \quad \text{-----------------(14)}$$

Note that there are a total of 2M possible values for each element of $\tau$. Therefore, the average value of these 2M data is computed and chosen as final reconstruction result in the simulation.

## III. NUMERICAL RESULTS

In this section, we report some numerical results obtained by computer simulations using the method described in the Section II . Let us consider a dielectric cylinder buried at a depth of $a$ = 0.1m in a lossless half space, as shown in Fig. 1. The permittivities in region 1 and 2 are characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.25\varepsilon_0$. The frequency of the incident waves is chosen to be 3 GHz and the number of illuminations is the same as that of cells. The incident waves are generated by numerous groups of radiators operated simultaneously.

The measurement is taken on a half circle of radius 0.5m about $(0, -a)$ at equal spacing. The number of measurement point is set to be 8 for each illumination. For avoiding trivial inversion of finite dimensional problems, the discretization number for the direct problem is four times that for the inverse problem in our numerical simulation.

In the example, the buried cylinder with 48 × 24 mm rectangular cross-sections of a perfectly conducting coated with dielectric materials with square cross-sessions is discretized into 16 × 8 cells, and the corresponding dielectric permittivities are plotted in Fig. 2. The model is characterized by simple step distribution of permittivity. Each cell has 3 ×3 mm cross-sections. The reconstructed permittivity distributions of the object are plotted in Fig. 3. The root-mean-square (RMS) error is about 2.7 %. It is apparent that the reconstruction is good.

For investigating the effect of noise, we add to each complex scattered field a quantity b+cj, where b and c are independent Gaussian random numbers having a Gaussian distribution over 0 to the noise level times the rms value of the scattered field. The noise levels applied include $10^{-5}$, $10^{-4}$, $10^{-3}$, $10^{-2}$, and $10^{-1}$ in the simulations. The numerical result for example is plotted in Fig. 4, respectively. They show the effect of noise is tolerable for noise levels below 1%.

## IV. CONCLUSIONS

Imaging algorithm for TE case is more complicated than that for the TM case, due to the added difficulties in the polarization charges. Nevertheless, the polarization charges cannot be ignored for this two-dimensional problem and all three-dimensional problems. In this paper, an efficient algorithm for imagine a complex cylinder, i.e., a conductor coated with dielectric materials, illuminated by TE waves, has been proposed. By properly arranging the direction of various unrelated waves, the difficulty of ill-posedness and nonlinearity is avoided. Thus, the permittivity distribution can be obtained by simple matrix operations. The moment method has been used to transform a set of integral equations into matrix form. Then these matrix equations are solved by the unrelated illumination method. Numerical simulation for imaging the permittivity distribution of a buried dielectric object has been carried out and good reconstruction has been obtained even in the presence of random noise in measured data. This algorithm is very effective and efficient, since no iteration is required.

## V.REFERENCES


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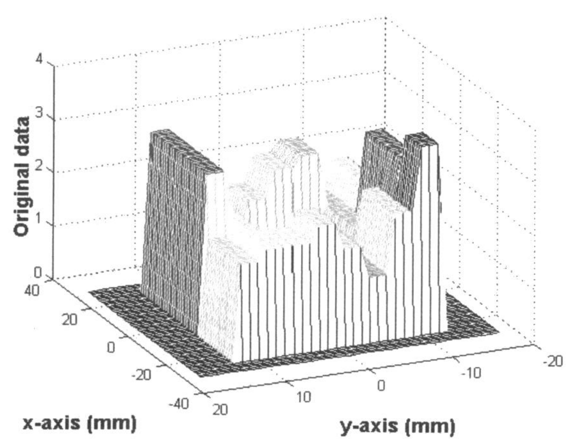


**Fig. 2** Original relative permittivity distribution.

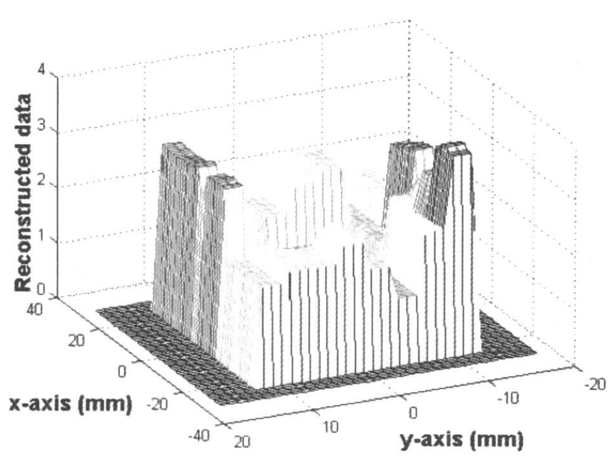


**Fig. 3** Reconstructed relative permittivity distribution.

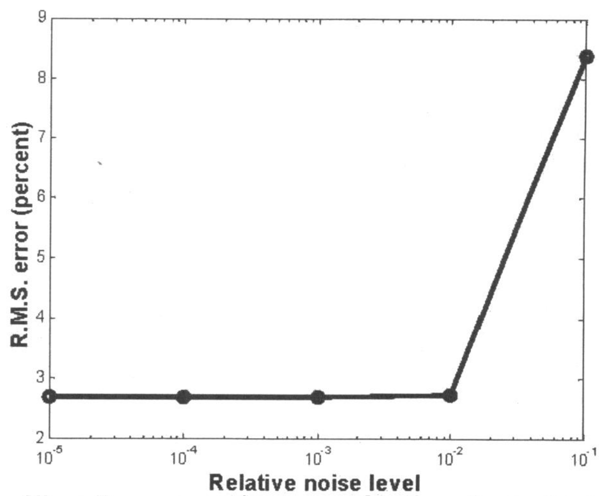


**Fig. 4** Reconstructed error as a function of noise level